Exercise 49

Find y' and y''.

$$y = \sqrt{1-\sec t}$$

Solution

Take the derivative using the quotient rule and the chain rule.

$$\begin{aligned}
y' = \frac{dy}{dt} &= \frac{d}{dt}\left(\sqrt{1-\sec t}\right) \\
&= \frac{1}{2}(1-\sec t)^{-1/2} \cdot \frac{d}{dt}(1-\sec t) \\
&= \frac{1}{2}(1-\sec t)^{-1/2} \cdot (-\sec t \tan t) \\
&= -\frac{\sec t \tan t}{2\sqrt{1-\sec t}}
\end{aligned}$$

Take another derivative.

$$\begin{aligned}
y'' = \frac{d}{dt}(y') = \frac{d}{dt}\left(-\frac{\sec t \tan t}{2\sqrt{1-\sec t}}\right) \\
&= -\frac{1}{2}\frac{d}{dt}\left(\frac{\sec t \tan t}{\sqrt{1-\sec t}}\right) \\
&= -\frac{1}{2}\frac{\left[\frac{d}{dt}(\sec t \tan t)\right]\sqrt{1-\sec t} - \left[\frac{d}{dt}(\sqrt{1-\sec t})\right](\sec t \tan t)}{1-\sec t} \\
&= -\frac{1}{2}\frac{\left\{\left[\frac{d}{dt}(\sec t)\right]\tan t + \sec t\left[\frac{d}{dt}(\tan t)\right]\right\}\sqrt{1-\sec t} - \left[\frac{1}{2}(1-\sec t)^{-1/2} \cdot \frac{d}{dt}(1-\sec t)\right](\sec t \tan t)}{1-\sec t} \\
&= -\frac{1}{2}\frac{\left[(\sec t \tan t)\tan t + \sec t(\sec^2 t)\right]\sqrt{1-\sec t} - \left[\frac{1}{2}(1-\sec t)^{-1/2} \cdot (-\sec t \tan t)\right](\sec t \tan t)}{1-\sec t} \\
&= -\frac{1}{2}\frac{(\sec t \tan^2 t + \sec^3 t)\sqrt{1-\sec t} + \frac{\sec^2 t \tan^2 t}{2\sqrt{1-\sec t}}}{1-\sec t} \\
&= -\frac{1}{2}\frac{\frac{2(\sec t \tan^2 t + \sec^3 t)(1-\sec t)}{2\sqrt{1-\sec t}} + \frac{\sec^2 t \tan^2 t}{2\sqrt{1-\sec t}}}{1-\sec t} \\
&= -\frac{1}{2}\frac{2(\sec t \tan^2 t + \sec^3 t)(1-\sec t) + \sec^2 t \tan^2 t}{2(1-\sec t)^{3/2}} \\
&= \frac{2\sec^4 t - 2\sec^3 t - 2\sec t \tan^2 t + \sec^2 t \tan^2 t}{4(1-\sec t)^{3/2}}
\end{aligned}$$